m; $F_{nq'}$, area of the boundary of the volume zones n and q'; g_{mn}, coefficients of convective heat transfer; N, total number of zones; M, total number of C zones; $\Delta\tau$, time step; T_n^i, zone temperature at time $\tau_i = \Delta\tau i$; $\Theta_k^{i+1/2}$, temperature field in the interval $[\tau_i, \tau_{i+1}]$; T_n^j, value of the zone temperature after the j-th iterations; $\| A \|$, a $(N - M) \times (N - M)$ matrix; $\vec{T}^i, \vec{P}, \vec{Q}, \vec{Q}^*$, $(N - M)$ dimensional vectors; β, positive constant, $\beta < 1$.

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SOLUTION OF THE STEADY-STATE PROBLEM OF HEAT EXCHANGE AND FLOW OF LUBRICANT IN RADIAL SLIDING BEARINGS WITH SELF-ALIGNING SEGMENTS

V. V. Rukhlinskii, L. A. Gura, and O. M. Borisenko

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The article describes a method based on the use of implicit finite-difference schemes, and it presents the results of the numerical solution of the problem of heat and exchange and flow of lubricant in multisegment radial sliding bearings.

The numerical solution of the problem of liquid flow and heat exchange in radial segmental sliding bearings is based on the well-known assumptions of Reynolds' hydrodynamic theory of lubrication. The physical parameters of oil were taken as constant, and they were determined according to the mean oil temperature in the gap, which was found from the solution of the heat-transfer equation.

The initial system of differential equations describing the intensity of heat transfer of the shaft in radial sliding bearings has the following form in dimensionless values:

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z}\right) = 6\frac{\partial h}{\partial x}, \tag{1}$$

$$u\frac{\partial t}{\partial x} + \left(\frac{v}{h} - u\frac{y}{h}\frac{\partial h}{\partial x}\right)\frac{\partial t}{\partial y} + w\frac{\partial t}{\partial z} = \frac{1}{\mathrm{Pe}h^2}\frac{\partial^2 t}{\partial y^2} + \frac{\mathrm{Ec}}{\mathrm{Re}h^2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right], \tag{2}$$

$$\mathrm{Nu} = \frac{R/L}{\pi\psi h(1 - t_f)}\int_0^{2\pi}\int_0^{L/R}\left.\frac{\partial t}{\partial y}\right|_{y=0} dxdz. \tag{3}$$

The coordinate Y is reckoned from the surface of the shaft, X from the horizontal axis, and Z from one of the end faces of the bearing.

The boundary conditions for solving the problem had the following form:

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for pressures

$$p(\varphi_{1j},\ 0<z<L/R) = p(\varphi_{2j},\ 0<z<L/R) = p_1,$$
$$p(x,\ 0) = p(x,\ L/R) = 0; \tag{4}$$

for speeds

$$u(x,\ 0,\ z) = 1,\ v(x,\ 0,\ z) = w(x,\ 0,\ z) = 0,$$
$$u(x,\ 1,\ z) = v(x,\ 1,\ z) = w(x,\ 1,\ z) = 0; \tag{5}$$

for temperatures

$$t(\varphi_{1j},\ y>0,\ z) = 0$$

or

$$t(\varphi_{1j},\ y,\ z) = 1 - 2y + y^2;\ \left.\frac{\partial t}{\partial x}\right|_{x=\varphi_{2j}} = \left.\frac{\partial t}{\partial x}\right|_{x=\varphi_{2j}-\Delta x};$$
$$t(x,\ 0,\ z) = 1;\ \left.\frac{\partial t}{\partial z}\right|_{z=0} = \left.\frac{\partial t}{\partial z}\right|_{z=\frac{L}{2R}} = 0;\ \left.\frac{1}{h}\frac{\partial t}{\partial y}\right|_{y=1} = \frac{q_1\delta}{\lambda(T_w - T_1)}. \tag{6}$$

It follows from expressions (4) that the pressure on the edges of the segments is taken as constant across the width of the bearing and equal to the initial pressure, and on the end faces as equal as zero. For finding the speeds, we use the conditions of adhesion and of the impermeability of solid boundaries (5).

Whereas the boundary conditions for pressures and the component of the speed are generally accepted, specifying the conditions of unambiguity for determining the temperatures is a very complex problem. Specifying an oil temperature that is constant across the thickness for $y > 0$ in the sections of the front edges is a characteristic trait of segments with individual lubricant supply. In this case the used hot oil in the gaps between the segments, under the effect of the centrifugal forces, is detached from the shaft surface, and the formation of the lubricant film occurs within the boundaries of each segment only by the newly supplied lubricant. The parabolic temperature distribution corresponds better to the design of bearings with intersegmental spaces flooded with oil, where the hot oil is not completely removed from the surface of the shaft; this affects the formation of the inlet profile of the temperatures in the subsequent segment.

It was assumed that in the end face sections of bearings, with $z = 0$ and $z = L/R$, convection is the sole mechanism of heat transfer in a liquid. With $y = 1$, the specific heat flow on the working surfaces of the segments was taken equal to a constant magnitude determined from the conditions of their cooling. With $x = \varphi_{2j}$, the temperature at the outlet edges was found from the condition of conserving the rate of its change in the vicinity of the boundary.

The projections of the speeds on the axes are determined from the expressions obtained by integrating the equations of motions taking into account the boundary conditions (5):

$$u = (1-y)\left(\frac{h^2}{2}\frac{\partial p}{\partial x}y + 1\right), \tag{7}$$

$$v = \frac{y^2h^2}{4}\frac{\partial p}{\partial x} - \frac{y^2h^2(2y-3)}{12}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2}\right) - \frac{y^2}{2}\frac{\partial h}{\partial x}, \tag{8}$$

$$w = \frac{h^2}{2}\frac{\partial p}{\partial z}y(1-y). \tag{9}$$

In the mathematical formulation of the problem, one of the principal questions is the specification of the distributions of the thickness of the lubricant according to segments. The load-bearing capacity, the damping properties, and the temperature state of the bearing depend on the distribution of the thicknesses, i.e., on the position of the segments relative to the shaft.

For the cylindrical bore of the segments and for the rigid shaft we write the distribution of the thicknesses of the lubricant layer in the adopted system of coordinates in the following way:

$$h = 1 + e\sin(x - \varphi_{\text{н}}) + \Delta h. \tag{10}$$

The value of Δh was determined from the condition that the minimum thickness is equal to $(1 - e)$ and is the same for all segments. The position of the section with minimum thickness for load-bearing segments was found from the condition of maximum load capacity, for the upper segments it was taken to coincide with the outlet edge of the segment. The maximum thickness of the film in the section of the inlet edge of the upper segments was equal to the value obtained for the cylindrical bore, i.e., from the condition $\Delta h = 0$.

The solution of Eqs. (1)-(3) was found in finite differences by the establishment method using locally unidimensional schemes [1].

The spatial network had 157 nodes along the x axis, 6 across the thickness of the lubricant layer, and 11 across the width of the bearing. With a view to the symmetry of the boundary conditions (4)-(6) relative to the central section, the distribution of pressures, speeds, and temperatures of the liquid lubricant was determined for half the bearing. The numerical solution of Eqs. (1) and (2) was effected by using implicit difference methods.

Approximation of the derivatives in the Reynolds equation was carried out by the three-point scheme which ensured absolute stability of the calculation.

The lack of derivatives of second order of the temperature with respect to the longitudinal coordinates in the heat transfer equation has the consequence that for their solution by an implicit scheme the remaining derivatives of first order cannot be replaced by the symmetric difference because then the conditions of positive approximation, which are indispensable for the stability of the method of matching, are not satisfied. The known methods of obtaining stable implicit difference schemes for solving boundary layer problems are very cumbersome, they require very fine space-time networks, and they lead to the loss of the principal advantage of implicit schemes: economy.

To overcome these difficulties, the approximation of the derivatives of first order in the i-th mode with respect to the coordinates x and z was effected by using difference combinations of the type

$$\left.\frac{\partial t}{\partial x}\right|_i = \frac{-3t_{i-1}+4t_i-t_{i+1}}{2\Delta x}\beta + \frac{3t_{i+1}-4t_i+t_{i-1}}{2\Delta x}(1-\beta), \tag{11}$$

where the terms are the derivatives in the (i – 1)-st and (i + 1)-st nodes written with weight β according to known three-point one-sided patterns, forward and backward, respectively, with an error $O(\Delta x^2)$.

Algebraic transformations yield the working formula

$$\left.\frac{\partial t}{\partial x}\right|_i = \frac{(1-4\beta)\,t_{i-1}-4\,(1-2\beta)\,t_i+(3-4\beta)\,t_{i+1}}{2\Delta x}. \tag{12}$$

The values of the weight may change from 0 to 1. For $\beta = 0.5$ we obtain the ordinary symmetric approximation with the smallest error $O(\Delta x^2)$. Depending on the sign of the coefficient in front of the derivative, the complex (12) satisfies the conditions of stability when $\beta \leqslant 0.25$ or $\beta \geqslant 0.75$. A change of β during the process of matching does not affect the stability of the solution.

The use of expression (12) makes it possible to obtain stable implicit schemes for solving difference analogs of differential equations of first and second orders by the method of matching. In that case, for derivatives of second order, an ordinary stable symmetric approximation with an error of $O(\Delta x^2)$ is maintained.

The calculation scheme using the method of matching presupposes the existence of two boundary conditions along each axis of coordinates including the X and Z axes, along which Eq. (2) is parabolic. This equation is approximate, it was obtained from a more accurate equation containing derivatives of second order with coefficients that were negligibly small compared with the coefficients of the derivatives of first order. A comparison of the numerical solutions of Eq. (2) and of the refined equation containing derivatives of second order with respect to the longitudinal coordinates showed that the results practically coincides with each other. On the other hand, the more complex equation entails unjustificable complications of the Algol program and longer computer time.

The use of the described method for solving the energy equation (2) confirmed that the given scheme is reliable and efficient, yet retains all the positive traits of implicit difference methods.

Equations (1)-(3) were solved successively for each segment. The position of the shaft in the bearing with specified eccentricity, characterized by the load angle, were found from the condition of equilibrium which had the form:

$$\int_0^{2\pi}\int_0^{L/R} p \sin x dz dx = \eta_{\text{n}}, \tag{13}$$

$$\int_0^{2\pi}\int_0^{L/R} p \cos x dz dx = 0. \tag{14}$$

The conditions of equilibrium separately for each segment served for determining the angular coordinate of its point of support (edge of oscillation)

TABLE 1. Results of the Calculations of Segmental Radial Sliding Bearings (T_1 = 50°C; T_W = 90°C; n = 3000 rpm; e = E/δ = 0.8)

No. of segments	α_{av}, W/m²·deg C	q_{av}, W/m²	T_{av}, °C	η_{H}	$\frac{1}{\psi}$	F	$G_{\Sigma T}$	φ_{H}	φ_{1j}	φ_{0j}	φ_{2j}	η_{Bj}	$\eta_{\Gamma j}$	h_{0j}
3	442	15 218	55,6	2,914	6,6189	19,2855	0,834	0,3792	0,7854 2,8797 4,974	1,801 3,726 5,7757	—2,3569 4,4506 6,5449	—1,2939 2,4356 1,7721	—0,4405 —3,1956 3,6486	1,6493 1,1117 0,3831
4	602	18 087	59,97	2,648	7,339	19,434	0,7266	—0,062166	0,1963 1,771 3,3379 4,9087	0,9628 2,5573 3,9391 5,4751	1,3744 2,9452 4,516 6,0868	—0,9669 —0,4238 2,0443 1,9948	0,6299 —0,5847 —1,9153 1,9874	1,2045 1,7842 0,5501 0,4427
5	811	24 232	60,14	1,84	11,46	21,09	0,515	0,16078	0,4713 1,7279 2,9845 4,2411 5,498	1,0841 2,3518 3,4412 4,7265 5,9306	1,4137 2,6703 3,9270 5,1836 6,4402	—0,7298 —0,3758 0,4980 1,9127 0,5364	3,3734 —0,3852 —1,5516 0,0464 1,5021	1,2445 1,8235 0,3171 0,4759 0,3047
6	667	19 812	60,32	1,742	10,15	17,822	0,475	—0,312665	0,1309 1,1781 2,2165 3,272 4,319 5,3669	0,6419 1,6991 2,727 3,734 4,7268 5,8127	1,1689 1,9635 3,0107 4,0579 5,1051 6,1522	0,3375 —0,3922 —0,2092 0,6909 1,4537 0,5359	0,4446 —0,0534 —0,4849 —1,009 0,03043 1,0783	1,3433 1,8106 1,4107 0,6937 0,4551 0,6075

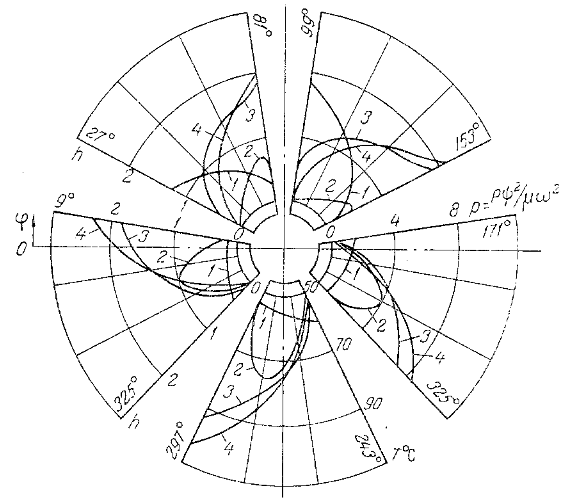

Fig. 1. Distributions of the relative thicknesses of the lubricant layer (1), of the dimensionless pressures (2) and temperatures on the working surfaces of the segments found for $t(\varphi_{1j}, y > 0, z) = 0$ (3) and $t(\varphi_{1j}, y, z) = 1 - 2y + y^2$ (4) in the central section of a five-segment bearing.

$$\varphi_{j\text{on}} = \varphi_{j1} + \arcsin \frac{\int\limits_{\varphi_{j1}}^{\varphi_{j2}} \int\limits_{0}^{L/R} p \sin (x - \varphi_{1j})\, dzdx}{\int\limits_{\varphi_{j1}}^{\varphi_{j2}} \int\limits_{0}^{L/R} p\, dzdx} . \tag{15}$$

During the process of solving the problem with a computer, it was envisaged that the consumption of lubricant would be determined for each segment

$$g_j = \frac{1}{6} \int\limits_{\varphi_{1j}}^{\varphi_{2j}} h^3 \frac{\partial p}{\partial z}\Big|_{z=0} dx + \frac{h^3}{12} \int\limits_{0}^{L/R} \frac{\partial p}{\partial x}\Big|_{x=\varphi_{2j}} dz + \frac{h}{2}\left(\frac{L}{R}\right) \tag{16}$$

and the frictional forces on the surface of the shaft

$$f_j = -\frac{1}{2} \int\limits_{\varphi_{1j}}^{\varphi_{2j}} \int\limits_{0}^{L/R} \frac{h^2 \frac{\partial p}{\partial x} + 1}{h}\, dzdx. \tag{17}$$

The integrals in (13)-(17) were calculated by methods of numerical integration.

With specified e, φ_{1j}, φ_{2j}, R, L, δ, ω, P_1, T_1, T_w, q_1 for the selected oil and the number of segments, we found the pressure, velocity, and temperature fields, the load angle, the load capacity coefficient, frictional losses, lubricant consumption, specific heat flows, and heat transfer coefficients.

The described method was applied to the calculation of three-, four-, five-, and six-segment bearings. The obtained results are presented in Table 1. In all cases the segments were arranged symmetrically about the vertical axis. The angular extent of the segments was determined from the expression $\gamma = 1.5\ \pi/k$, where k is the number of segments. The shaft diameter was taken to be 0.5 m, the width of the bearing 0.4 m, and there was not heat flow on the working surfaces of the segments. The physical parameters of the lubricant were found from the exponential dependences obtained by approximation of the tabulated data of [2] for oil "Turbinnoe-22." As the characteristic temperature for determining the physical parameters in the energy equation and in the dimensionless integral characteristics we took the mean flow rate temperature of the oil in the bearing. Such an approach yields acceptable agreement between calculation and experimental data, as was previously demonstrated in [3] for radial bearings with 360° contact.

A comparison of the results of the numerical investigations shows that bearings with self-aligning segments have a larger load capacity with eccentricities $e > 0.85$ than solid bearings [3], and a smaller load capacity when the eccentricity is

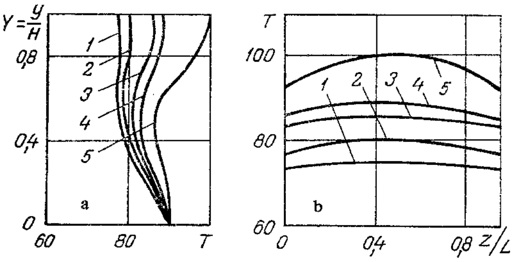

Fig. 2. Temperature distribution at the outlet from the fourth segment of a five-segment bearing across the thickness of the lubricant layer in the central section (a) and across the width of the segment on its working surface (b) in dependence on the rotational frequency of the shaft: 1) n = 1000 rpm; 2) 1500; 3) 2500; 4) 3000; 5) 5000. T, °C.

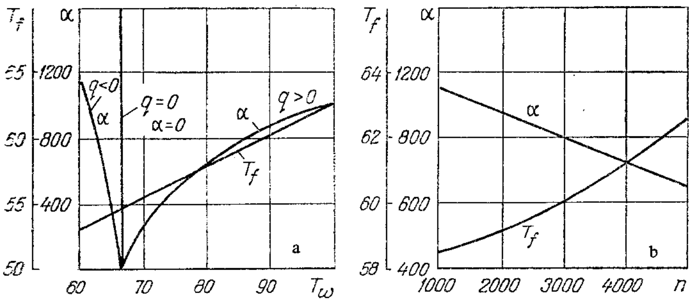

Fig. 3. Dependences of the averaged heat transfer coefficients and oil temperatures in a five-segment bearing on the temperature of the shaft journal (a) and on the rotational frequency (b). T_w, °C; n, rpm; α, $W/m^2 \cdot °C$; T_f, °C.

is small. This indicates that they have good damping properties and high efficiency under heavy loads.

When the frictional losses in ordinary and in multisegment radial sliding bearings are to be compared, the flow regimes of the lubricant must be taken into account. If the oil flow is laminar in both types of bearing, then the solid bearings are more economical. The values of F in Table 1 exceed the frictional losses in solid bearings by 40-50%. This is due to the appearance of zones of elevated hydrodynamic pressures on the upper segments and the associated additional losses of power. It therefore makes sense to use segmental bearings when in solid bearings there are no more possibilities of laminarizing flow and increase their damping properties by improved design.

The distributions of the relative thicknesses of the lubricant film, of the dimensionless pressure and temperature values of the working surface of the segments in the central section of a five-segment bearing are presented in Fig. 1.

It can be seen from a comparison of the results of solving the heat transfer equation with different boundary conditions at the entrance to a segment that when the temperature distribution across the thickness of the lubricant on the inlet edges of the segments is parabolic, the maximum temperatures in the gaps are 3-7° higher than when $t(\varphi_{1j}, y > 0, z) = 0$ is specified. This shows that it is advisable to arrange for removal of the hot oil ahead of the entrance to the segment.

Increased circumferential speed of the shaft leads to increased oil temperature in the working gaps of the segments and to lower intensity of heat transfer on the surface of the shaft (Fig. 2). The same effect occurs when the load is increased.

An important parameter characterizing the thermal state of the bearing is the surface temperature T_w of the shaft which is introduced as a specified magnitude into the formulation of the problem, but which in reality is dependent on the operating regime of the bearing and on the external conditions of heat supply to the shaft. Study of the effect of T_w on the intensity of heat transfer of the shaft (Fig. 3) showed that there exists such a shaft temperature (which is usually higher than the oil temperature at the inlet to the bearing) at which the directions of the heat flows on the surface of the shaft change. For instance, for the calculated five-segment bearing (Fig. 2) at T_w = 66.8°C the heat transfer coefficient and the mean specific heat flow are equal to zero. As the positive direction of the heat flow we took the heat supply from the shaft to the oil.

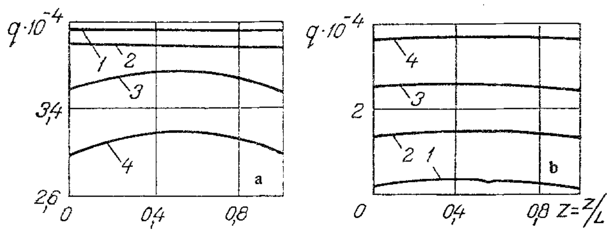

Fig. 4. Distributions of the specific heat flows (W/m^2) averaged over the circumference of a five-speed bearing in dependence on the circumferential speed (a: 1) 1500 rpm; 2) 2000; 3) 3000; 4) 5000) and on the shaft temperature (b: 1) 70°C; 2) 80; 3) 90; 4) 100).

The distributions of the mean specific heat flows across the width of a five-segment bearing in dependence on the circumferential speed and the surface temperature of the shaft are presented in Fig. 4.

Thus, the use of the described method of numerical investigation of heat exchange and flow of lubricant makes it possible to carry out a detailed qualitative and quantitative analysis of the intensity of the thermophysical processes occurring in liquid-friction bearings with self-aligning segments with a view to the principal regime and geometric parameters of their operation.

NOTATION

X, Y, Z, running coordinates; x, y, z, dimensionless coordinates; L, width of the segment; R, radius of the shaft; H, running thickness of the lubricant; $h = H/\delta$, dimensionless thickness of a layer of lubricant; δ, mean radial gap; $u = U/\omega R$; $v = V/\omega\delta$; $w = W/\omega R$, dimensionless speeds in the directions of the x, y, z axes, respectively; P, $p = P\psi^2/\mu\omega$, dimensional and dimensionless pressure at an arbitrary point; $\psi = \delta/R$, mean relative gap; P_1, oil pressure at the inlet to a segment; T, running temperature; T_1, T_2, T_W, oil temperature at the inlet to a segment, at the drain from the bearing, and on the surface of the shaft, respectively; T_f, mean flow-rate temperature; $t = (T - T_1)/(T_W - T_1)$, dimensionless oil temperature at an arbitrary point; $Nu = \alpha d/\lambda$, Nusselt number; $Pe = \omega\delta^2/a$, Peclet number; $Re = \omega\nu^2/\nu$, Reynolds number; $Ec = \omega^2R^2/C_p(T_w - T_1)$, Eckert number; α, mean heat-transfer coefficient; C_p, specific heat of the oil; ρ, density of the oil; λ, thermal conductivity; ν, coefficient of kinematic viscosity; a, thermal diffusivity; φ_H, load angle; φ_{1j}, φ_{2j}, φ_{0j}, angular coordinates of the beginning, the end, and the support of the j-th segment, respectively; q_1, mean specific heat flow on the working surface of a segment; $e = E/\delta$, relative eccentricity; Δh, change in the thickness of the lubricant in comparison with the cylindrical bore due to a turn of the segment induced by hydrodynamic forces; Q_H, vertical load on the bearing; G_j, $g_j = G_j/\rho\omega\delta R$, mass and dimensionless flow rates of lubricant, respectively, through the gap of the j-th segment; F_j, $f_j = F_j\psi/\mu\omega R^2$, dimensional and dimensionless friction force, respectively; $\eta_H = Q_H\psi^2/\mu\omega RL$, summary load capacity coefficient of the bearing; η_{Bj}, η_{rj}, dimensionless projections of the resulting forces of the pressures on the vertical and horizontal axes of the j-th segment, respectively.

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